Extended $d_{x^2-y^2}$-wave superconductivity.
Flat nodes in the gap and the low-temperature asymptotic properties of high-T_c superconductors

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Abstract. Remarkable anisotropic structures have been recently observed in the order parameter $\Delta_{\mathbf{k}}$ of the underdoped superconductor $Bi_2Sr_2CaCu_2O_{8+\delta}$. Such findings are strongly suggestive of deviations from a simple $d_{x^2-y^2}$-wave picture of high-T_c superconductivity, *i.e.* $\Delta_{\mathbf{k}} \sim \cos k_x - \cos k_y$. In particular, flatter nodes in $\Delta_{\mathbf{k}}$ are observed along the $k_x = \pm k_y$ directions in $\mathbf{k}$-space, than within this simple model for a d-wave gap. We argue that nonlinear corrections in the $\mathbf{k}$-dependence of $\Delta_{\mathbf{k}}$ near the nodes introduce new energy scales, which would lead to deviations in the predicted power-law asymptotic behaviour of several measurable quantities, at low or intermediate temperatures. We evaluate such deviations, either analytically or numerically, within the interlayer pair-tunneling model, and within yet another phenomenological model for a d-wave order parameter. We find that such deviations are expected to be of different sign in the two cases. Moreover, the doping dependence of the flatness of the gap near the nodes is also attributable to Fermi surface effects, in addition to possible screening effects modifying the in-plane pairing kernel, as recently proposed.

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1 Introduction

Power laws in the low-temperature asymptotic behaviour of several linear response electronic properties provide complementary evidence for d-wave symmetry of the order parameter (OP) $\Delta_{\mathbf{k}}$ of high-T_c superconductors [1,2] as well as preliminary evidence of 'exotic' shapes in the OP of heavy fermion superconductors, such as UPt_3 [3]. This has to be contrasted to an "activated" behaviour $\propto \exp(-\beta\Delta_{\min})$, appropriate of s-wave superconductors, or, in the case of mixed symmetry, of superconductors with a non-vanishing s-wave contribution to their OP, where $\Delta_{\min} = \min_{\mathbf{k}} |\Delta_{\mathbf{k}}| > 0$. In the case of a non-empty nodal manifold for the superconducting excitation spectrum $E_{\mathbf{k}}$, defined as the locus of points in $\mathbf{k}$-space such that $E_{\mathbf{k}} = 0$, a large number of quasiparticles can be created near such nodes, thus dominating all the low-temperature electronic properties [4]. An exact analysis allows one to relate the exponent of the leading power of the low-T expansion of a given linear response function to the dimension of the

Fermi manifold (defined as the locus of states in $\mathbf{k}$-space with vanishing dispersion relative to the Fermi level in the normal state, $\xi_{\mathbf{k}} = 0$) and the topological nature of the nodal manifold, *viz.* a collection of points, of line segments, or of surface patches [5,1].

On the basis of group theoretical arguments, the simplest choice for a d-wave gap function on a square lattice is $\Delta_{\mathbf{k}} = \Delta g(\mathbf{k})$, where Δ is a T-dependent parameter, and

$$g(\mathbf{k}) = \frac{1}{2}(\cos k_x - \cos k_y) \qquad (1)$$

is the first basis function associated with the d-wave irreducible representation of the appropriate crystal point group, C_{4v} [1]. We remark that $g(\mathbf{k})$ is generated, together with an extended s-wave term proportional to

$$h(\mathbf{k}) = \frac{1}{2}(\cos k_x + \cos k_y), \qquad (2)$$

by a nearest-neighbour interaction term in real space. Here and in the following we shall measure the wavevectors in units of the appropriate inverse lattice spacings. Proportionality to equation (1) allows $\Delta_{\mathbf{k}}$ to vanish linearly at a

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given point along the Fermi line, which for most cuprate superconductors can be modelled by the tight-binding expansion:

$$\xi_{\mathbf{k}} = -2t(\cos k_x + \cos k_y) + 4t' \cos k_x \cos k_y - \mu = 0, \quad (3)$$

where $t = 0.25$ eV, $t' = 0.45t$ measure nearest and next-nearest neighbour hopping, respectively, and μ is the chemical potential.

On the other hand, increasing experimental evidence above all from angle-resolved photoemission spectroscopy (ARPES) suggests a richer structure in $\mathbf{k}$-space for the OP of the underdoped high-T_c superconductor $Bi_2Sr_2CaCu_2O_{8+\delta}$ [6]. In particular, the superconducting gap near the nodal points turns out to be flatter than predicted by the simple assumption $\Delta_{\mathbf{k}} \propto g(\mathbf{k})$ [6]. Such a feature is consistent with the observation of whole ungapped segments of the Fermi line above T_c in the pseudogap regime of underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$ [7], and will of course serve as a constraint for a microscopic understanding of the pairing mechanism.

Quite remarkably, qualitatively similar deviations from a $g(\mathbf{k})$-like dispersion have been evidenced in the $\mathbf{k}$-dependence of the antiferromagnetic gap in the related insulating compounds $X_2CuO_2Cl_2(X = Ca, Sr)$ [8]. Such a finding has been interpreted in terms of an interrelation between the antiferromagnetic phase of the parent insulator and the underdoped regime of the intervening superconductor [9].

In this paper, we argue that such extended structures in the superconducting OP, interpolating between point and line nodes, can be included in the definition of $\Delta_{\mathbf{k}}$ as higher order terms in $g(\mathbf{k})$. We shall then look for their signatures in the low-temperature asymptotic electronic properties of the superconducting cuprates, as corrections to the predicted power-law behaviour. In deriving our results analytically, we will specifically consider the interlayer pair-tunneling (ILT) mechanism of high-T_c superconductivity [10], which has been shown to accurately reproduce most of the observed gap features [11].

2 Extended d-wave gap within the ILT model

A distinguishing feature of the ILT mechanism, compared to other proposed models of HTSC, is that superconductivity is driven by a gain in kinetic, rather than potential, energy as temperature is lowered below the critical temperature T_c. It is assumed that coherent single particle hopping between adjacent CuO_2 layers in the cuprates is suppressed by the non-Fermi liquid character of the normal state (*e.g.* due to spin-charge separation), while interlayer coherent tunneling of Cooper pairs is allowed as soon as a superconducting condensate is established. Confined coherence [12] within CuO_2 layers in the normal state is indeed largely motivated by the absence of coherent transport along the c-axis, whereas a comprehensive theoretical understanding of it is still lacking. However, there is now abundant experimental evidence that c-axis

transport in the normal state indeed is incoherent, while that in the superconducting state may not be [13]. This seems to warrant attention being paid to unconventional models of high-T_c superconductivity based on relieving c-axis frustrated kinetic energy. Recent findings [14,15] suggest, however, that the ILT mechanism alone is not sufficient to account for the large condensation energy E_c, as extracted experimentally from measurements of the penetration length λ_c of several single layered compounds, such as $Tl_2Ba_2CuO_{6+\delta}$ [14,15], whereas the predictions of the ILT model agrees with the measured value of E_c for $La_{2-x}Sr_xCuO_4$ [16–19]. It has been pointed out, however, that while considerable experimental effort has been devoted to the determination of λ_c, extracting E_c from existing data on electronic specific heat is by no means straightforward [20]. A direct evaluation of E_c from its mean-field expression at $T = 0$ [21] would relieve the complications arising from thermal fluctuation effects, inherent in the method of integrating specific heat data, from $T = 0$ through T_c, recently pointed out in reference [20]. By utilizing the gap equation (Eq. (4)), within the ILT model, and the expression relating λ_c at $T = 0$ to E_c [22], we find results for $\lambda_c(T = 0)$ in Bi2212 which are within factors of order unity from the experimental values, rather than factors of 10 to 20 [23]. The observed doping dependence of λ_c [17] is also qualitatively reproduced [23].

The emerging scenario suggests therefore that some in-plane effective interaction might co-operate with the ILT mechanism in establishing the superconducting state [24]. One could think of such a mechanism as a seed for the Cooper instability, and the origin of the gap's dominant d-wave symmetry. Once Cooper pairs are formed in the appropriate symmetry channel(s) *via* such in-plane effective interaction, the ILT mechanism would allow the condensate for an additional energy gain, by releasing the constraint of in-plane segregation.

Without explicitly specifying the microscopic origin of the in-plane mechanism, we therefore assume the in-plane pairing potential to be given by $V_{\mathbf{k}\mathbf{k}'} = Vg(\mathbf{k})g(\mathbf{k}')$ $(V < 0)$, thus allowing for d-wave symmetry of the order parameter. The issue of the competition with other subdominant (s-wave) symmetry channels in the presence of ILT has been addressed in reference [11], showing that the d-wave contribution wins out at optimal doping and in the underdoped regime. Despite its kinetic nature, ILT can be absorbed in the interacting part of the Hamiltonian as an effective term $T_J(\mathbf{k})\delta_{\mathbf{k}\mathbf{k}'}$, whose $\mathbf{k}$-space locality enforces in-plane momentum conservation during a tunneling process [10]. Following reference [10], we assume $T_J(\mathbf{k}) = t_\perp^2(\mathbf{k})/t \equiv T_J g^4(\mathbf{k})$, being $t_\perp(\mathbf{k})$ the single-particle interlayer hopping amplitude, with $t_\perp(\mathbf{k}) \propto g^2(\mathbf{k})$, as suggested by ARPES as well as by band structure calculations [10,25]. A standard mean-field diagonalization technique then yields the following expression for the energy gap [26,11]:

$$\Delta_{\mathbf{k}} = \frac{\Delta g(\mathbf{k})}{1 - T_J(\mathbf{k})\chi_{\mathbf{k}}}, \quad (4)$$

where $\chi_{\mathbf{k}} = (2E_{\mathbf{k}})^{-1}\tanh(\beta E_{\mathbf{k}}/2)$ is the superconducting pair susceptibility, and $E_{\mathbf{k}} = (\xi_{\mathbf{k}}^2 + |\Delta_{\mathbf{k}}|^2)^{1/2}$ is the upper branch of the superconducting elementary excitation spectrum.

Along the Fermi line ($\xi_{\mathbf{k}} = 0$) at $T = 0$, one immediately sees that:

$$\Delta_{\mathbf{k}} = \Delta g(\mathbf{k}) + \frac{1}{2}T_{\mathrm{J}}g^4(\mathbf{k})\mathrm{sgn}[g(\mathbf{k})]. \quad (5)$$

Such an expression, together with manifestly fulfilling the requirement of *d*-wave symmetry, also endows the superconducting gap with a richer structure near the nodal points along the $k_x = \pm k_y$ directions. This is probably best seen by considering the Fourier expansions:

$$g(\mathbf{k}) = -2\sum_{m=1}^{\infty} J_{4m-2}(k)\cos[(4m-2)\phi], \quad (6\mathrm{a})$$

$$h(\mathbf{k}) = J_0(k) + 2\sum_{m=1}^{\infty} J_{4m}(k)\cos(4m\phi), \quad (6\mathrm{b})$$

$$T_{\mathrm{J}}(\mathbf{k})/T_{\mathrm{J}} = \frac{9}{64} + \frac{a_0}{2} + \sum_{m=1}^{\infty} a_{4m}\cos(4m\phi), \quad (6\mathrm{c})$$

with

$$a_{4m} = \frac{1}{32}J_{4m}(4k) + \frac{1}{2}J_{4m}(2k) + \frac{3}{16}(-1)^m J_{4m}(2k\sqrt{2})$$
$$-\frac{3}{4}(-1)^m J_{4m}(k\sqrt{2}) - \frac{1}{4}J_{4m}\left(\frac{k}{\sin\phi_0}\right)\cos(4m\phi_0). \quad (7)$$

Here, the generic wavevector $\mathbf{k}$ is expressed in terms of its modulus k and of the angle ϕ formed with the ΓX direction in the first Brillouin zone (1BZ), $\mathbf{k} = (k\cos\phi, k\sin\phi)$, $J_\alpha(x)$ are Bessel functions of the first kind and order α, and $\tan\phi_0 = \frac{1}{3}$.

Equation (5) is to be contrasted to the phenomenological fit

$$\Delta_{\mathbf{k}} = \Delta[B\cos(2\phi) + (1-B)\cos(6\phi)] \quad (8)$$

proposed in reference [6] for $\Delta_{\mathbf{k}}$ along the Fermi line: instead of requiring an in-plane interaction extended to further neighbours, equation (4) endows the superconducting gap with the observed flat structure around the nodes, through the ILT term $T_{\mathrm{J}}(\mathbf{k})$. In Figure 1, we fit equation (5) against Mesot *et al.*'s experimental data for one of the underdoped Bi2212 samples in reference [6], having $T_c = 75$ K. A remarkable agreement follows already by fixing Δ so that $|\Delta_{\mathbf{k}}|$ reproduces the maximum datum at $\mathbf{k} = (\pi, \pi)$, whereas T_{J} is taken to be 0.04 eV [10]. In particular, besides obtaining an enhanced maximum value of $|\Delta_{\mathbf{k}}|$ at $\mathbf{k} = (\pi, \pi)$, we are thus able to recover the anomalously flat region around the node at $\phi = 45°$ in a rather natural way. We note, however, that our fit requires $\Delta \approx T_{\mathrm{J}}/2$ around optimal doping, which will not be without consequences in evaluating other fundamental quantities [23].
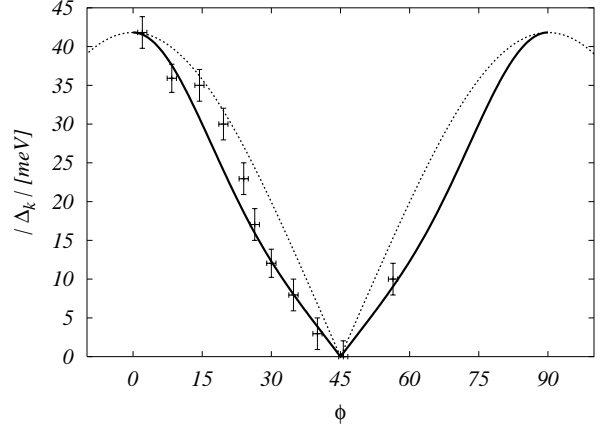

Fig. 1. Fit for $|\Delta_{\mathbf{k}}|$ within the ILT model, equation (5) (solid line), and in the case of a simple *d*-wave gap $|\Delta_{\mathbf{k}}| = \Delta g(\mathbf{k})$ (dashed line), against Mesot *et al.*'s ARPES data for underdoped Bi2212 ($T_c = 75$ K, Ref. [6]).

Equation (5) already contains the doping dependence of the observed gap anisotropy, although in a hidden way. As pointed out in reference [11], the auxiliary parameter Δ is to be self-consistently determined by solving the appropriate gap equation. Besides being intrinsically doping dependent, this equation is unconventionally modified by the presence of a $\mathbf{k}$-local effective interaction, as induced by the ILT mechanism. Moreover, the role of the contribution $\propto T_{\mathrm{J}}g^4(\mathbf{k})$ in equation (5) is strongly influenced by the actual location of the Fermi line, as $g^4(\mathbf{k})$ is sharply peaked at $\mathbf{k} = (0, \pi)$ (and symmetry related points).

Equation (5) also facilitates the evaluation of the slope of the superconducting gap $v_\Delta = (1/2)\mathrm{d}|\Delta_{\mathbf{k}}|/\mathrm{d}\phi$ at the nodal point along the Fermi line. Such a quantity is related to the temperature derivative of the superfluid stiffness at $T = 0$. In particular, it is seen that the ratio $v_\Delta/\Delta_{\max}$ decreases with underdoping [6]. From equation (5), one derives that v_Δ is independent of T_{J}, and that therefore a doping induced change of v_Δ through Δ essentially can be traced back to the actual position of the Fermi line, as discussed above, within the ILT model. The ratio $v_\Delta/\Delta_{\max}$ will anyway deviate from its value within simple *d*-wave (BCS-like) models, as a function of doping, due to the enhancement of $\Delta_{\max}$ induced by ILT.

3 Low-temperature asymptotic behaviour of electronic properties

We now address the issue, whether such extended features of the OP near the nodes, as those described in the previous section, induce deviations in the low or intermediate temperature asymptotic behaviour of linear response electronic properties in the superconducting state. In what follows, we shall limit our discussion to clean superconductors, and neglect impurity effects altogether. Mean-field (BCS or BCS-like) expressions for most linear response electronic properties are available also in the case of anisotropic, *i.e.* non *s*-wave, superconductors.

In particular, we have in mind observable quantities such as the superconducting density [27], the electronic specific heat [27], the spin susceptibility [27], the penetration depth [28], the thermal conductivity [29], and so on. Their expressions basically involve the evaluation of some integral of the kind:

$$\mathcal{F}[\beta; \varphi_{\mathbf{k}}(\beta)] = \frac{1}{(2\pi)^2} \int d^2\mathbf{k}\,\varphi_{\mathbf{k}}(\beta)e^{-\beta E_{\mathbf{k}}}, \qquad (9)$$

where $\beta = (k_{\mathrm{B}}T)^{-1}$, $\varphi_{\mathbf{k}}(\beta)$ is a (dimensional) function of wavevector $\mathbf{k}$ and temperature, related to the electronic quantity of interest, and the integration is extended to the 1BZ, $\mathbf{k} \in [-\pi, \pi] \times [-\pi, \pi]$ (see Appendix A). In the case of d-wave superconductors, $E_{\mathbf{k}}$ is allowed to vanish at the intersection between the Fermi line and the nodal lines of the gap function. Around such points, quasiparticles can be created in large numbers. In the limit of low temperatures ($\beta \to \infty$), therefore, the value of the integral in equation (9) is dominated by the contributions from wavevectors $\mathbf{k}$ close to such point nodes. Around such nodes, it is useful to introduce the new sets of coordinates (k_1, k_2) or (ϵ, θ), defined as [4]:

$$\xi_{\mathbf{k}} \simeq \mathbf{v}_{\mathrm{F}} \cdot \mathbf{k} \equiv v_{\mathrm{F}}k_1 = \epsilon\cos\theta, \qquad (10\mathrm{a})$$
$$\Delta g(\mathbf{k}) \simeq \mathbf{v}_2 \cdot \mathbf{k} \equiv v_2 k_2 = \epsilon\sin\theta, \qquad (10\mathrm{b})$$

in units where $\hbar = 1$. Here, v_{F} and v_2 are the Fermi velocity and a suitable "gap" velocity, respectively, evaluated at $E_{\mathbf{k}} = 0$, and ϵ measures the distance in energy from a given dispersionless point implicitly defined by $E_{\mathbf{k}} = 0$. In terms of the new coordinates, the superconducting spectrum for a simple d-wave superconductor near a node therefore looks like an anisotropic Dirac cone [4],

$$E_{\mathbf{k}} \sim \left(v_{\mathrm{F}}^2 k_1^2 + v_2^2 k_2^2\right)^{1/2} = \epsilon. \qquad (11)$$

The observation of flatter structures near the nodes [6] not only implies a more significant anisotropy ratio v_{F}/v_2, but also the possibility that higher order terms in ϵ may contribute to $E_{\mathbf{k}}$, (Eq. (11)). Indeed, within the ILT model, from equation (4) at $T = 0$ one obtains

$$E_{\mathbf{k}} \sim \epsilon\left[1 + \left(\frac{\epsilon}{\epsilon_{\star}}\right)^3 \sin^6\theta\right], \qquad (12)$$

to lowest order in $\epsilon/\epsilon_{\star}$, with $1/\epsilon_{\star}^3 = (1/2)(T_{\mathrm{J}}/\Delta^4)$ related to the pair-tunneling amplitude T_{J} and to the auxiliary gap parameter Δ (Figs. 2 and 3).

Other models, based on extended in-plane pairing mechanisms, would in general yield different polynomial corrections in ϵ to $E_{\mathbf{k}}$. For instance, within the spin fluctuation theory [30], the following phenomenological expansion holds for the momentum distribution of the superconducting energy gap [31]

$$\Delta_{\mathbf{k}} = \Delta g(\mathbf{k}) \sum_{n=0}^{N} d_n h^n(\mathbf{k}), \qquad (13)$$

with all coefficients $d_n = 1$. We explicitly observe that for $N = 0$, $d_0 = 1$, one recovers the simple d-wave gap
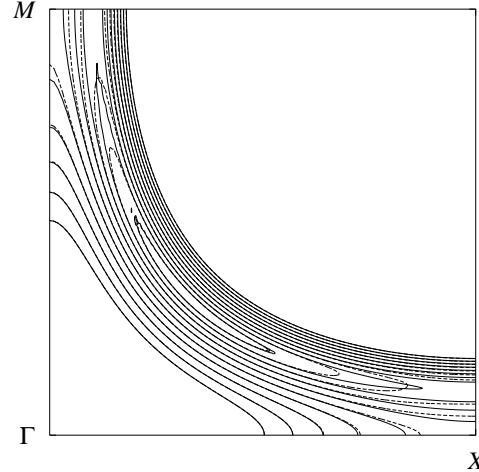

Fig. 2. Typical contour lines of the superconducting spectrum $E_{\mathbf{k}}$ in the simple d-wave case (dashed lines) and in presence of ILT (continuous line).
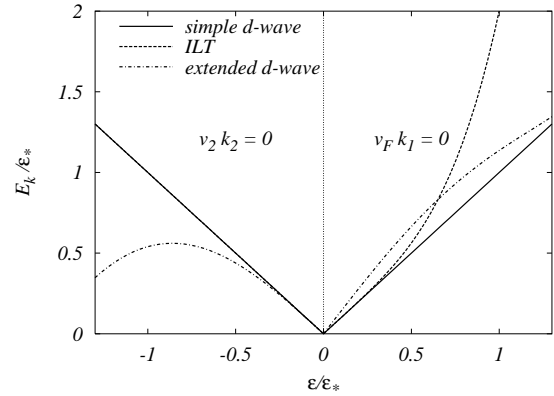

Fig. 3. Deviations from the simple d-wave case, equation (11) (continuous line), of the superconducting spectrum $E_{\mathbf{k}}$ around a node, within the ILT model, equation (12) (dashed line), and in the case of an extended d-wave gap, equation (15) (dashed-dotted line), as a function of the reduced coordinates $\epsilon/\epsilon_{\star}$ ($\epsilon/\tilde{\epsilon}_{\star}$, respectively). Such deviations are most easily seen along the direction of the nodal line (left panel, $v_2 k_2 = 0$ or $\theta = \pi$), and along the Fermi line (right panel, $v_{\mathrm{F}}k_1 = 0$ or $\theta = \pi/2$). Note that along $v_2 k_2 = 0$, one has $E_{\mathbf{k}} = \epsilon$ also within the ILT model.

$\Delta_{\mathbf{k}} \propto g(\mathbf{k})$, while the case $N = 1$, with the identifications $\Delta \mapsto B\Delta$, $d_0 = 1$, $d_1 = 4(1 - B)/B$, maps to [6,31]:

$$\Delta_{\mathbf{k}} = \Delta[Bg(\mathbf{k}) + (1 - B)g(2\mathbf{k})], \qquad (14)$$

which is compatible with the phenomenological fit equation (8) proposed by Mesot *et al.* in reference [6] for their experimental data of $|\Delta_{\mathbf{k}}|$ along the Fermi line [6,31]. In particular, equation (14) would follow from a correction $\delta V_{\mathbf{kk'}} \propto g(2\mathbf{k})g(2\mathbf{k'})$ to the in-plane coupling, corresponding to next-nearest neighbours interaction.

In such a particular case, and assuming for simplicity $t' = \mu = 0$ in equation (3), one straightforwardly obtains

$$E_{\mathbf{k}} \sim \epsilon\left[\cos^2\theta + \sin^2\theta\left(1 - \frac{\epsilon}{\tilde{\epsilon}_{\star}}\cos\theta\right)^2\right]^{1/2}, \qquad (15)$$

where $\tilde{\epsilon}_\star = tB/(1-B)$ is now related to the ratio of near-est *vs.* next-nearest neighbours coupling. Therefore, both within the ILT model and within other models, based on extended in-plane pairing, the additional mechanism responsible for the nonlinear correction to $E_{\mathbf{k}}$ away from its nodes introduces new energy scales (here, $\epsilon_\star$ or $\tilde{\epsilon}_\star$, respectively). Figure 3 depicts the two different ways in which $E_{\mathbf{k}}$ deviates from the cone-like shape, (Eq. (11)), near a node, in the two cases given by equations (12) and (15).

In the absence of any such additional mechanism ($\epsilon_\star, \tilde{\epsilon}_\star = 0$), the leading contribution to equation (9) for the simplest, reference case $\varphi_{\mathbf{k}}(\beta) \equiv 1$ is:

$$\mathcal{F}_1(\beta) \equiv \mathcal{F}[\beta; 1] \doteq \frac{A}{\beta^2}, \qquad (16)$$

where $A = (2\pi v_{\mathrm{F}} v_2 \Delta)^{-1}$ is a doping-dependent factor, and $\doteq$ denotes equality up to terms vanishing exponentially with β at all energy scales, as $\beta \to \infty$ ($T \to 0$). Equation (16) should be regarded as typical of the power-law asymptotic low-temperature behaviour of the super-conducting electronic properties within a simple d-wave BCS-like model.

In order to obtain an asymptotic expansion for $\mathcal{F}_1(\beta)$ as $\beta \to \infty$ ($T \to 0$), including the corrections due to ILT, (Eq. (12)), we observe that the integration over ϵ in equation (9) is actually made of two contributions:

$$\int_0^\infty \mathrm{d}\epsilon = \int_0^{\epsilon_\star} \mathrm{d}\epsilon + \int_{\epsilon_\star}^\infty \mathrm{d}\epsilon. \qquad (17)$$

In the first integral, we may safely retain only the linear term $E_{\mathbf{k}} \sim \epsilon$ in the exponent, since $\epsilon \le \epsilon_\star$. In the second contribution, this is no longer possible, and equation (12) has to be retained in full. However, since $\epsilon \ge \epsilon_\star > 0$, one can make use of Laplace's (saddle point) method for the integral over angles around $\theta = 0$. The final result is:

$$\begin{aligned}
\mathcal{F}_1(\beta) &\sim \frac{A}{\beta^2}\Big[1 - (1 + \beta\epsilon_\star)\mathrm{e}^{-\beta\epsilon_\star} \\
&\quad + \frac{1}{3\pi}\Gamma\Big(\frac{1}{6}\Big)\beta\epsilon_\star \Gamma\Big(\frac{4}{3}, \beta\epsilon_\star\Big)\Big] \\
&\sim \frac{A}{\beta^2}\Big[1 - \Big(1 + \beta\epsilon_\star \\
&\quad - \frac{1}{3\pi}\Gamma\Big(\frac{1}{6}\Big)(\beta\epsilon_\star)^{5/6}\Big)\mathrm{e}^{-\beta\epsilon_\star}\Big], \qquad (18)
\end{aligned}$$

where $\Gamma(x)$, $\Gamma(\alpha, x)$ are Euler gamma and incomplete gamma functions, respectively [32]. A comparison of equations (16) and (18) is provided by Figure 4, and shows that $\mathcal{F}_1(\beta)$ gets effectively suppressed in the presence of flat nodes in the order parameter, as provided by the ILT mechanism, with respect to the simple d-wave case, at an energy scale $\sim \epsilon_\star$.

No such simple asymptotic expansion for $\mathcal{F}_1(\beta)$ is available in the extended d-wave case described by equation (15), and the integrations have to be performed numerically. Figure 4 shows the result, with the identifications $A \mapsto \tilde{A} = (2\pi v_{\mathrm{F}} v_2 B \Delta)^{-1}$ and $\epsilon_\star \mapsto \tilde{\epsilon}_\star$. In this case, equation (15) provides $E_{\mathbf{k}}$ with a different kind of
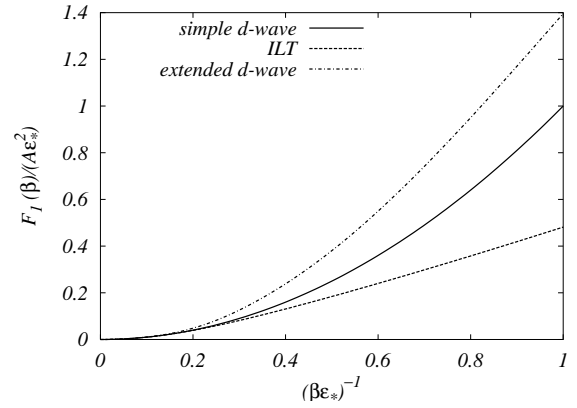

Fig. 4. Asymptotic power-law ($\propto T^2$, solid line) and modified power-law behaviours of $\mathcal{F}_1(\beta)$, in the presence of ILT (dashed line), and in the extended d-wave case (dashed-dotted line), as $(\beta\epsilon_\star)^{-1} \to 0$. Given the values used for the fits of $|\Delta_{\mathbf{k}}|$ along the Fermi line in reference [6] and in Figure 1, it turns out that $\epsilon_\star \sim 250$ K.

anisotropy with respect to the simple case, (Eq. (11)), than equation (12) does. While in the latter case one always has $E_{\mathbf{k}} \ge \epsilon$, here one has $E_{\mathbf{k}} \gtrless \epsilon$, depending on the angle θ (*cf.* Fig. 3). As a consequence, $\mathcal{F}_1(\beta)$ is enhanced with respect to the simple d-wave case, at an energy scale $\sim \tilde{\epsilon}_\star$.

4 Conclusions

Motivated by recent experimental findings of extended flat structures in the order parameters of the underdoped d-wave superconductor $\mathrm{Bi_2Sr_2CaCu_2O_{8+\delta}}$ [6], we have addressed the issue of whether nonlinear, high-energy corrections to the superconducting energy spectrum $E_{\mathbf{k}}$ around the gap nodes induce deviations in the predicted power-law behaviour of several electronic properties at low or intermediate temperatures. We have shown that nonlinear corrections to $E_{\mathbf{k}}$ in general introduce additional energy scales in the problem. Deviations from the usual power-law behaviour of the superconducting electronic properties are indeed to be expected at such energy scales, but the actual value and sign of such deviations are specific to the model under consideration. In particular, within the ILT model, we have explicitly derived the expected corrections to a typical power-law asymptotic behaviour as $T \to 0$, showing these to be negative, whereas within a phenomenological model of extended d-wave superconductivity [6] such corrections are predicted to be positive. Whether such deviations will actually be observable in real measurements of superconducting electronic properties, will of course depend on the effective values of the additional energy scales $\epsilon_\star$ or $\tilde{\epsilon}_\star$ in real compounds.

Appendix A: Low-temperature superconducting electronic properties

We now give a sketch of how the low-temperature asymptotic behaviour of several electronic properties of interest can be reduced to that of $\mathcal{F}_1(\beta)$ or its derivatives. Most electronic quantities in the superconducting state are in fact given by equation (9), with $\varphi_{\mathbf{k}}(\beta)$ actually depending on $\mathbf{k}$ only through $E_{\mathbf{k}}$. In what follows, $f(\epsilon) = [1 + \exp(\beta\epsilon)]^{-1}$ denotes the Fermi function.

Within BCS theory, the electronic specific heat is given by [27]:

$$
\begin{aligned}
C_V &= \sum_{\mathbf{k}} 2k_{\mathrm{B}}\beta E_{\mathbf{k}} \left[E_{\mathbf{k}} + \frac{\partial E_{\mathbf{k}}}{\partial \beta} \right] \left(-\frac{\partial f}{\partial E_{\mathbf{k}}} \right) \\
&\sim \sum_{\mathbf{k}} 2k_{\mathrm{B}}\beta E_{\mathbf{k}}^2 \left(-\frac{\partial f}{\partial E_{\mathbf{k}}} \right) \\
&\sim 2k_{\mathrm{B}}\beta^2 \mathcal{F}_1''(\beta),
\end{aligned}
\tag{A.1}
$$

where apices denote derivatives with respect to β. Here, $\varphi_{\mathbf{k}}(\beta) = 2k_{\mathrm{B}}\beta^2 E_{\mathbf{k}}^2$, and we have made use of the fact that $(-\partial f/\partial E_{\mathbf{k}}) \doteq \beta \exp(\beta E_{\mathbf{k}})$.

Analogously, the unrenormalized, static, isotropic spin susceptibility $\chi_0 = \chi_0(\mathbf{q} \to 0, \omega \to 0)$, which is directly related to the Knight shift, is simply given by [21,33]:

$$
\chi_0 = \sum_{\mathbf{k}} \left(-\frac{\partial f}{\partial E_{\mathbf{k}}} \right) \doteq \beta \mathcal{F}_1(\beta).
\tag{A.2}
$$

The expression of the electronic thermal conductivity for an anisotropic d-wave superconductor also involves an average of $(-\partial f/\partial E_{\mathbf{k}})$ over the 1BZ [29,34]:

$$
\kappa_{\mathrm{e}} = \frac{1}{T} \sum_{\mathbf{k}} \left(-\frac{\partial f}{\partial E_{\mathbf{k}}} \right) E_{\mathbf{k}}^2 \left(\frac{\partial E_{\mathbf{k}}}{\partial k_x} \right)^2 \tau(\mathbf{k}),
\tag{A.3}
$$

where $\tau(\mathbf{k})$ is the superconducting quasiparticles lifetime. Due to the presence of the x component of the group velocity $\nabla_{\mathbf{k}} E_{\mathbf{k}}$, however, its expression in our notation reduces to:

$$
\kappa_{\mathrm{e}} = \frac{1}{8\pi^2} \frac{\ell_0}{v_2} \frac{1}{T} \int_0^\infty \mathrm{d}\epsilon\, \epsilon^3 \int_0^{2\pi} \mathrm{d}\theta \left(-\frac{\partial f}{\partial \epsilon} \right) \left(\cos\theta + \frac{v_2}{v_{\mathrm{F}}} \sin\theta \right)^2,
\tag{A.4}
$$

where $\ell_0 = v_{\mathrm{F}}\tau(\mathbf{k}_{\mathrm{F}})$ is the quasiparticle mean free path at the nodes. The final result crucially depends on the anisotropy ratio v_2/v_{F}, and would be different in the two cases given by equations (12, 15), due to their different θ dependence. This has to be contrasted with the result obtained in the simple d-wave case, where [34]:

$$
\kappa_{\mathrm{e}} = \eta k_{\mathrm{B}}^3 T^2 \frac{\ell_0}{v_2} \left(1 + \frac{v_2^2}{v_{\mathrm{F}}^2} \right),
\tag{A.5}
$$

with $\eta = (8\pi)^{-1} \int_0^\infty \mathrm{d}x\, x^3(-\partial f/\partial x)$.

Appendix B: A limiting case

In the absence of in-plane coupling, a spurious solution of the mean-field gap equation at $T = 0$ can be implicitly expressed via [11]:

$$
E_{\mathbf{k}} = \frac{1}{2} T_{\mathrm{J}}(\mathbf{k}) = \frac{1}{2} T_{\mathrm{J}} g^4(\mathbf{k}).
\tag{B.1}
$$

In such a limiting case, the superconducting energy spectrum would have purely kinetic origin, and would be identified with the interlayer pair-tunneling amplitude, divided by two. A closed expression can then be obtained for $\mathcal{F}_1(\beta)$, by utilizing the useful result:

$$
\frac{1}{(2\pi)^2} \int \mathrm{d}^2\mathbf{k}\,\mathcal{G}[\eta(\mathbf{k})] = \frac{2}{\pi^2} \int_{-1}^1 \mathrm{d}x\,\mathcal{G}(x) K(\sqrt{1-x^2}),
\tag{B.2}
$$

where $\mathcal{G}[\eta(x)]$ is any continuous functional of $\eta(\mathbf{k}) = h(\mathbf{k})$ or $g(\mathbf{k})$ alone, and $K(x')$ ($x' = \sqrt{1-x^2}$) is the complete elliptic integral of first kind [32]. From equation (9), expanding $K(x')$ around $x = 0$, one eventually arrives at the closed expression:

$$
\mathcal{F}_1(\beta) \sim \frac{1}{\pi^2} \Gamma\left(\frac{1}{4} \right) \frac{1}{\zeta^{1/4}} \left(2\log 2 - \frac{1}{4}\psi\left(\frac{1}{4} \right) + \log \zeta^{1/4} \right),
\tag{B.3}
$$

where $\psi(x)$ is the digamma function [32], and the ILT amplitude T_{J} here fixes the appropriate energy scale, through $\zeta = \frac{1}{2}\beta T_{\mathrm{J}}$.

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